PPP Projects Valuation with Managerial Flexibility

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Abstract

Infrastructure projects are vital and essential for sustainable development of countries. Most governments are facing issues of shortage of fiscal budgets, which make these governments impossibly to provide full financial support for all important infrastructure projects. With considering ample financial funds and managerial competent of private sectors, governments introduce eagerly the desirable model of public–private partnership for the implementation of infrastructure projects. Successful public-private partnership projects may create more social-economic activities in societies, and make the substantial progress of society for development. However, unsuccessful public-private partnership projects may lead to waste of society resources. Based on the experience on the implementation of public-private partnership projects in Taiwan, we discover that the failure projects occurred at construction or operation stages instead of project negotiation stage. A fair allocation of project risk may assure successful implementation of the projects and attract competent investors to invest in infrastructure projects as well. In fact, infrastructure projects bear with huge public benefits. Governments should play more aggressive roles to assist private sectors in implementing the projects successfully. Governments could choose to provide sufficient incentives to investors for investing in non-viable public-private partnership projects in order to reduce the project risk for private investors. The incentives, considered in this study, are to provide managerial flexibility to investors. A real option model for analyzing effects of governmental strategies on the valuation of Public-Private-Partnership (PPP) projects is established for investigating financial effects of incentives in this study. A university dormitory project is considered as an empirical study. The results from case study reveal that the project financial benefits increase for providing incentives by government to investors. These results may provide useful information for government in project negotiation, which can lead to an all-win project negotiation with enhancement of project financial feasibility for non-viable projects and successful project implementation for public private partnership projects.

Keywords: Public private partnership, Real option, Risk analysis, Project uncertainty.

1. Introductions

For public goods that generate positive externalities, such as pollution reduction, social development, and the like, the market price does not reflect the true value of those goods because there is no market for the externalities. Producers cannot be compensated for generating positive externalities and under-producing or not producing the goods. Economists justify the government's intervention to correct the market failure by claiming that the aim of such intervention is to increase the economy's growth rate by internalizing the external benefits (Lucas, 1988; Romer, 1986). Lump–sum subsidy is one of the measures that governments take to finance the intervention effectively (Devarajan et al., 1998). For financially non-viable but economically viable PPI project, government subsidy is used to compensate the nonrecovered financial cost to generate the potential economic benefit. Our paper proposes an approach to determine an optimal level of subsidy to generate maximum social benefit from PPI project.

Researches have applied the real options on the valuation of PPP projects in the literatures in last two decades, such as Trigeorgis (1991, 1993, 1996). Trigeorgis argued that investors do not consider the capital expenditure of PPP projects has the nature, which can be late execution or termination, in conventional investment analysis. In addition, the PPP projects are also exposed under uncertainty due to the long term project life. The convention investment analysis uses the discounted cash flow (DCF) approach to evaluate project value. The DCF approach does not adjust timely in considering the value or re-evaluation of project parameters in dynamic market environment. Managers of the project may consider the expansion of the project scale in case of promising market conditions for maximizing the project profits. They also may turn down or scale down the project in case of bad market conditions. In other words, every project has a potentially unlimited profits and with limited lost. Trigeorgis call this expanded net present value (NPV), which is sum of conventional NPV and the value of real option, is the managerial flexibility on projects. Trigeorgis and Mason (1987) proposed the following formula:

$$\text{Expanded NPV} = \text{Passive NPV} + \text{Real option value}$$

The real option provide investors a novel thinking on project valuation with dynamic market conditions, which is called managerial flexibility. The real option theory derives from financial option theory. However, the financial option is used as a hedge measures to deal with uncertainty on underling assets. The managerial flexibility is to give project managers the right to determine the time to invest and the scale of project base on the market conditions.

Trigeorgis (1996) uses the real option model to evaluate the power plant project with managerial flexibility and demonstrates that postpone or turn down the project does have positive project value. Ho and Liu (2002) derives a binominal model of real option (BOT-OV) to assess the project financial feasibility and set up the options in case of project bankruptcy. They also analyses the financial effects on project value with government guarantees and option in project negotiations. Rose (1998) analyses an Australia high speed rail project with contract conditions that the government may terminate the concession agreement, when the after tax internal rate of return is greater than a certain preset ratio. He estimates the return value of project in early return of project to government and value of project postpone for Project Company.

Bowe and Lee (2004) study the real option value of postpone, termination, scale down project of project in operation and construction period in a binomial real option model of Taiwan high speed rail project. Cheah and Liu (2006) consider the government guarantee as a put option and apply the Monte Carlo simulation to value the put option of the project in their study. Huang and Chou (2006) use the minimum revenue guarantee (MRG) as a European option and combine the abandon option in construction stage into a compound option. They find that increasing MRG may reduce the value of abandon option. Once MRG reaches a certain value, the value of abandon option drops to zero. Alonso-Conde, Brown and Rojo-Suarez (2007) use an inter-city highway project in Melbourne to find incentive effect of investment to investors by providing the investors the right to delay payments on loyalty and permitting the government to exercise the early transfer of facility. Liu and Cheah (2009) consider government guarantee and subsidy with presetting tariff ceiling price as real option in project negotiation in a water pollution treatment plant project in southern China.

The managerial flexibility to project investors consists investment decisions on the time to invest and the scale to expand. This right of managerial flexibility and other business is considered as a call option in this study. A call option is a financial security which gives its owner the right (but not the obligation) to buy an underlying project for a pre-specified value (this is the fixed benchmark called the strike or exercise value, k) on (or before) the expiration date (T) of the option contract. A "European" call option can be exercised only on the expiration date whereas an American call can be exercised at any time up to and including the expiration date.

After Black and Scholes (1972) published their path-breaking paper providing a model for valuing dividend-protected European options in 1972, real option pricing theory has made vast strides. Black and Scholes considered a "replicating portfolio" — a portfolio composed of the underlying asset and the risk-free asset that had the same cash flows as the option being valued— to derive their final formulation. While their derivation is quiet complicated in mathematics, there is a simpler binomial model for valuing real options that draws on the same logic. We extend the discrete model into a continuous model. Furthermore, we conduct the sensitivity analysis on the continuous real option model.

2. The Binomial Model

Assume that project value, V, follows the standard geometric Brownian motion or the standard geometric Wiener process. The binomial real option-pricing model is based upon a simple formulation for the project value process, in which the project value can move to one of two possible values in any time period. Assuming that the project value follows a multiplicative binomial process over discrete periods. The rate of return on the project over each period can have two possible values: $u-1$ with probability q, or $d-1$ with probability $1-q$. Thus, if the current project value is V_0, the project value at the end of the period will be either uV_0 or dV_0. We can represent this movement with the following diagram (Black and Scholes, 1972):

$$V_0 = \begin{cases} uV_0 & with\ \ probability\ \ q \\ dV_0 & with\ \ probality\ \ 1-q \end{cases}$$

V_0 is the current project value; the value moves up to uV_0 with probability p and down to dV_0 with probability 1-p in any time period. In a multi-period binomial process, the valuation has to proceed iteratively; i.e., starting with the last time period and moving backwards in time until the current point in time.

C_u is the option value of the underling project with project value increasing to uV_0.

$$C_u = max[0, uV_0 - K] \qquad 1 \leq u < \infty \tag{3.1}$$

C_d is the option value of the underling project with project value decreasing to dV_0.

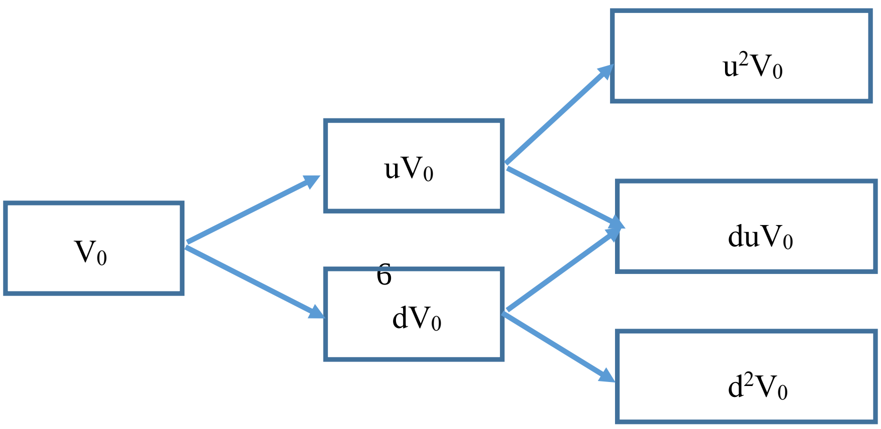
$$C_d = max[0, dV_0 - K] \qquad -\infty < d \leq 1 \tag{3.2}$$

K is the exercise value, which is an agreement value on the expiration date in an option.

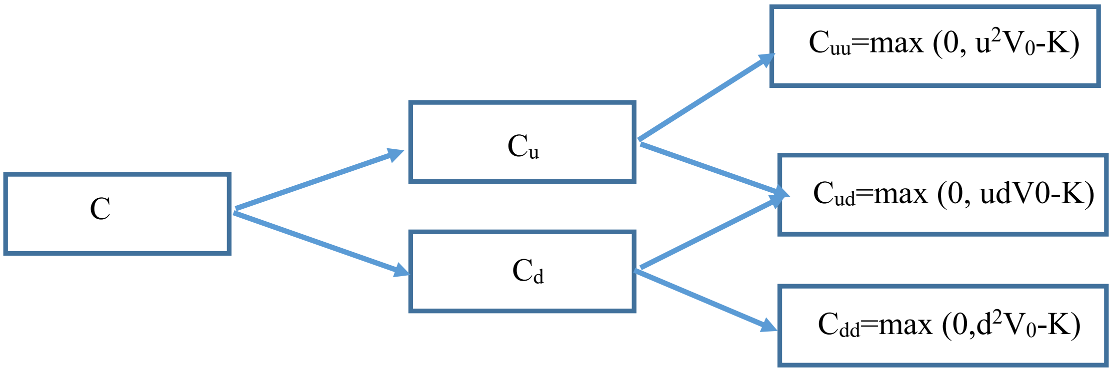
c

$$= \begin{cases} C_u = max[0, uV_0 - K] & project\ \ value\ \ increases\ \ with\ \ propability\ \ \ p \\ C_d = max[0, dV_0 - K] & project\ \ value\ \ decreases\ \ with\ \ propability\ \ 1-p \end{cases}$$

Consider the fluctuation of project value in two consecutive periods as follows:



The option value becomes the following:



In n consecutive periods, the option value has n+1 possible outcomes. The expected value of the option in present value is as follows:

$$C = \left[\sum_{j=0}^{n}\left(\frac{n!}{j!(n-j)!}\right)p^{j}(1-p)^{n-j}max\left[0, u^{j}d^{n-j}V_0 - K\right]\right]/r^{n} \qquad (3.3)$$

Where n is the period number. j is the number of period which project value increases. n-j is the number of period which project value decreases. r is risk-free rate of return.

For all $j < a$,

$$max\left[0, u^{j}d^{n-j}V_0 - K\right] = 0$$

And for all $j \geq a$,

$$max\left[0, u^{j}d^{n-j}V_0 - K\right] = u^{j}d^{n-j}V_0 - K$$

$$a \equiv \text{ the smallest non} - \text{negative integer such that}$$
$$\geq log(K/V_0 d^n)/\, log(u/d)$$

By breaking up C into two terms, we can rearrange the equation

$$C = V_0 \left[\sum_{j=a}^{n} \left(\frac{n!}{j!(n-j)!}\right) p^j (1-p)^{n-j} \left(\frac{u^j d^{n-j}}{r^n}\right)\right] - Kr^{-n} \left[\sum_{j=a}^{n} \left(\frac{n!}{j!(n-j)!}\right) p^j (1-p)^{n-j}\right]$$

The binomial model provides insight into the determinants of project value. The value of an option is not determined by the expected value of the project but by its current value, which, of course, reflects expectations about the future. The cash flows on the two positions offset each other, leading to no cash flows in subsequent periods. The option value also increases as the time to expiration is extended, as the value movements (u and d) increase, and with increases in the interest rate.

3. The Real Option Continuous Model for Single Project

The binomial model is a discrete model, and can be extended into a continuous model. We rewrite the equation into the form below:

$$C = V_0 \varphi_1[a\ ; n, p'] - Kr^{-n} \varphi_2[a\ ; n, p]$$

The first item in the right hand side of the equation (3.5) is

$$\varphi_1[a\ ;\ n, p'] = \sum_{j=a}^{n} \left[\frac{n!}{j!\,(n-j)!}\right] p^j (1-p)^{n-j} \left(\frac{u^j d^{n-j}}{r^n}\right)$$

$\varphi_1[a \quad ; n, p']$ is expected value of project value increment for those end project value great than the strike project value. Let p′≡(u/r)p and 1–p′≡(d/r)(1–p), we find

$$\varphi_1[a \ ; \ n, p'] = \sum_{j=a}^{n} \left[\frac{n!}{j!\,(n-j)!} (p')^j (1-p')^{n-j}\right]$$

The second item in the right hand side of the equation (3.5) is

$$\varphi_2[a \ ; \ n, p] = \sum_{j=a}^{n} \left[\frac{n!}{j!\,(n-j)!} p^j (1-p)^{n-j}\right]$$

$\varphi_2[a \ ; \ n, p]$ is the possibility to exercise on the expiration date.

A “European” call option can be exercised only on the expiration date whereas an American call can be exercised at any time up to and including the expiration date. The binomial model can be used to simulate the American option, which can exercise at any time before expiration date T. In case of large n, which represents large numbers of periods, we can then derive the limiting case of the equation. This leads to a continuous model and can be used to simulate the European option. The detailed derivation is shown in appendix B.

$$\varphi[a \ ; n, p'] \to N(x)$$
$$\varphi[a \ ; n, p] \to N\left(x - \sigma\sqrt{t}\right)$$

Where N is standard normal accumulative distribution function. x is the following

$$x \equiv \frac{log \frac{V_0}{Kr^{-t}}}{\sigma\sqrt{t}} + \frac{1}{2}\sigma\sqrt{t}$$

Finally, we can derive the continuous model as follows:

$$C = V_0 N(x) - K r^{-t} N\left(x - \sigma\sqrt{t}\right)$$

In case of $K=V_0$, we can obtain

$$\frac{C}{V_0} = N(x) - r^{-t} N\left(x - \sigma\sqrt{t}\right)$$

While the binomial model provides an intuitive feel for the determinants of option value, it requires a large number of inputs, in terms of expected future values at each node. The continuous model is not an alternative to the binomial model; rather, it is one limiting case of the binomial.

The binomial model is a discrete-time model for project value movements, including a time interval (t) between value movements. As the time interval is shortened, the limiting distribution, as t approaches 0, can take one of two forms. If as t approaches 0, value changes become smaller, the limiting distribution is the normal distribution and the value process is a continuous one. If as t approaches 0, value changes remain large, the limiting distribution is the Poisson distribution, i.e., a distribution that allows for price jumps. The continuous model applies when the limiting distribution is the normal distribution, and it explicitly assumes that the value process is continuous and that there are no jumps in project value.

The version of the real option continuous model is designed to value European real options, which were dividend-protected. Thus, neither the possibility of early exercise nor the payment of dividends affects the value of options in this model.

The value of a call real option in the continuous model can be written as a function of the following variables:

V_0 = Current value of the underlying project

K = Strike value of the option

t = Life to expiration of the option

r = Riskless interest rate corresponding to the life of the option

σ^2 = Variance in the log (value) of the underlying project

The model for value of a call real option can be written as:

$$C = V_0 N(d_1) - Ke^{-rt} N(d_2)$$

Where

$$d_1 = \frac{log\left(\frac{V_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

The process of valuation of the real option model involves the following steps:

Step1: The inputs to the real option model are used to estimate d_1 and d_2.

Step2: The cumulative normal distribution functions, $N(d_1)$ and $N(d_2)$, corresponding to these standardized normal variables are estimated.

Step3: The present value of the exercise value is estimated, using the continuous time version of the present value formulation:

Present value of exercise value = $K\ e^{-rt}$

Step4: The value of the call is estimated from the continuous real option model.

The determinants of value in the model are the same as those in the binomial - the current value of the project, the variability in project value, the time to expiration on the option, the strike value, and the riskless interest rate.

4. Sensitivity analysis

A sensitivity analysis of equation (3.8) is conducted in order to study the influence of parameters on the value of a real option of projects. A call option is a financial security which gives its owner the right (but not the obligation) to buy an underlying project for a pre-specified value (this is the fixed benchmark called the strike or exercise value, k) on (or before) the expiration date (T) of the option contract. A put option gives its owner the right (but not the obligation) to sell an underlying project at the strike value on or before the expiration of the put option contract. The managerial flexibility in projects are considered as call options. The variables or parameters in equation (3.8) are shown in Table 1.

Table 1: The five variables or parameters in real option model

Variables or Parameters	Definition
V_0	The current value of projects
$K=V_T$	The exercise value of projects
r	Risk free interest rate (%)
σ	Standard deviation in the log (value) of the underlying project
T	Life to expiration of the option (in years)

"Greek letters" are defined as the sensitivities of the option value to a single-unit change in the value of either a state variable or a parameter. Such sensitivities can represent the different dimensions to the risk in an option. Government who sell option to investors of the projects can manage their risk by Greek letters analysis.

In this section, the definitions and derivations of Greek letters is discussed. We derive Greek letters for call options on non-dividend stock.

Delta (Δ)

The delta of an option, Δ, is defined as the rate of change of the option value respected to the rate of change of underlying project value:

$$\Delta = \frac{\partial C}{\partial V_0}$$

where C is the option value and V_0 is underlying project value.

Theta (Θ)

The theta of an option, Θ, is defined as the rate of change of the option value respected to the passage of time:

$$\Theta = \frac{\partial C}{\partial t}$$

where C is the option price and t is the passage of time.

If $\tau = T - t$, theta (Θ) can also be defined as minus one timing the rate of change of the option value, C, respected to time to maturity. The derivation of such transformation is easy and straight forward:

$$\Theta = \frac{\partial C}{\partial t} = \frac{\partial C}{\partial \tau}\frac{\partial \tau}{\partial t} = (-1)\frac{\partial C}{\partial \tau}$$

where $\tau = T - t$ is time to maturity. For the derivation of theta for various kinds of option, we use the definition of negative differential on time to maturity.

Gamma (Γ)

The gamma of an option value, Γ, is defined as the rate of change of delta respected to the rate of change of underlying project value:

$$\Gamma = \frac{\partial \Delta}{\partial V_0} = \frac{\partial^2 C}{\partial V_0^2}$$

where C is the option value and V_0 is the underlying project value.

Because the option is not linearly dependent on its underlying project, delta-neutral hedge strategy is useful only when the movement of underlying project value is small. Once the underlying project value moves wider, gamma-neutral hedge is necessary.

Vega (ν)

The vega of an option, ν, is defined as the rate of change of the option value respected to the volatility of the underlying project:

$$\nu = \frac{\partial C}{\partial \sigma}$$

where C is the option price and σ is volatility of the project value.

Rho (ρ)

The rho of an option is defined as the rate of change of the option value respected to the interest rate:

$$\rho = \frac{\partial C}{\partial r}$$

where C is the option value and r is interest rate. The rho for an ordinary call option should be positive because higher interest rate reduces the present value of the strike value which in turn increases the value of the call option. Similarly, the rho of an ordinary put option should be negative by the same reasoning.

We show results of the sensitivities of the option value to the change in the value of state variables or parameters in Table 2 and Table 3. Risk management is one of the important topics in finance today, both for academics and practitioners. Given the recent credit crisis, one can observe that it is crucial to properly measure the risk related to the ever more complicated financial projects.

Table 2: Formula for sensitivity analysis of a real option

No.	Risk factors	Formula
1	Δ	$N(d_1)$
2	Γ	$\frac{1}{V_0 \ \sigma\sqrt{T}} \ N' \ (d_1)$
3	υ	$V_0\sqrt{T}N' \ (d_1)$
4	ρ	$KTe^{-\gamma T}N(d_2)$

5	θ	$-\frac{V_0 \sigma}{2\sqrt{T}} N'(d_1) - \gamma K e^{-\gamma T} N(d_2)$

The sensitivity analysis shows the effects of the parameters on the call value of real option. The results are summarized in Table 3.

5. Parameters for case study

A BOT project of dormitory in National United University is considered as a case study for application of real option model. Some parameters are assumed in the following. Concession period of project is 40 years, which consists of 2 years of construction period and 38 years of operation period. Inflation rate is assumed 2% annually, which is average value of inflation rate in Taiwan in 2002~2011. The expenditure in operation period includes mountainous fee, miscellaneous fee, and insurance fee. Rent and salary are adjusted annually by 2% per year. These assumptions are based by the feasibility study of the BOT dormitory projects in the university.

5.1 Construction cost of the project

Construction expenditure of the dormitory project is 408,844,293 NT$. Unit cost is 86,281 NT$ per Ping. This cost includes direct cost and indirect cost. Direct cost consists five parts of expenditure, which are structure cost 246,402,000 NT$, finishing and facility fee 28,431,000 NT$, electrical and mechanical facilities fee 56,862,000NT$, land scape fee 18,954,000 NT$, and sanitary facilities fee 24,400,000NT$. Indirect costs include 8 items, which are project management fee (assumed 1.5% direct cost) 5,625,735 NT$, soil condition investigation fee 1,500,000 NT$, design fee (assumed 4.5% direct cost) 16,877,205 NT$, construction management fee (assumed 1% direct cost) 3,750,490 NT$, land development loyalty 1,000,000 NT$, commence fee (0.5% of direct cost and indirect cost) 2,014,012 NT$, land lease fee 27,458 NT$, and insurance fee (assumed 0.4% of direct cost) 3,000,392 NT$.

Three type of bedroom, which are single bed room, two-bed room, and four-bed room in the dormitory project. There are 140 single bedrooms with 4 Pings per room, 300 two-bed rooms with 5.3 Pings per room, and 170 four-bed rooms with 8 Pings per room. Total room number is 610 with 1420 beds. Public floor area is 35% in total floor area with 4,738.5 Pings.

5.2 Operation cost of dormitory project

The major income of dormitory project is the rent collections from residences. Rent is 5,000 NT$ per month for one-bed room, 6,500 NT$ per month for two-bed room, and 9,000 NT$ for four-bed room. Assume that 90% of rooms are leased. Students pay rent for 10 month per year. Annual revenue is 40,964,000 NT$. The operation cost is divided into two portions, fixed expenditure and variable expenditure. Variable expenditures are personnel fee, facility fee, maintenance fee, miscellaneous fee, operation loyalty, business tax, building tax, and land tax, which is 2,000,000 NT$ (assumed 20% of annual revenue in total). The total operation expenditure is 10,192,800 NT$ annually.

With the data above, we can show the annual cash flows in Figure 4.1 and accumulate cash flow in Figure 4.2. Cash flows, which slightly drop at years 14, 17, 26, 32, and 38, are due to replacement expenditure. Replacement fee occurs for finishing for every 12 years and for electrical and mechanical facilities for every 17 years. Figure 4.2 shows that the payback year (PB) is 18 years and 31 years for discounted cash flows.

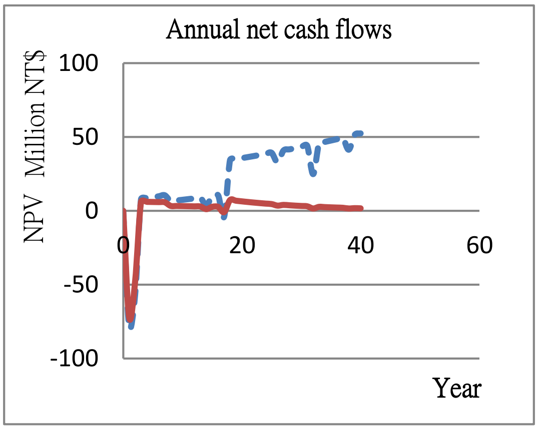


Figure 5.1 Annual cash flows of the project

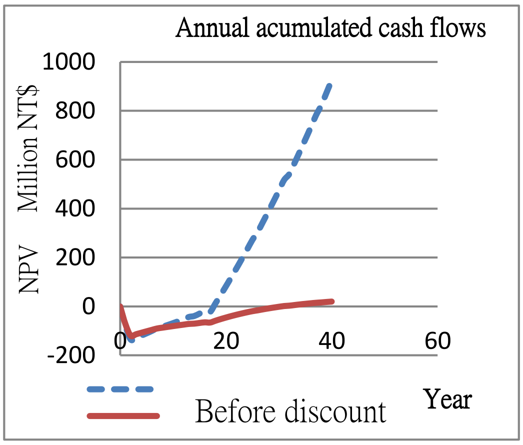


Figure 5.2 Accumulated cash flows of the project

5.3 Scenario analysis of dormitory project

We conduct a scenario analysis for dormitory project with various bed numbers. Four cases are considered. Case 1 is dormitory with 710 beds. Construction expenditure is 205,689,625 NT$. NPV is 1,687,097, IRR is 9.16%, and PB is 38 years. Case 2, which is the base case, is dormitory with 1420 beds. Construction expenditure is 408,844,293 NT$. NPV is 20,101,236, IRR is 9.99%, and PB is 31 years. Case 3 is dormitory with 2,130 beds. Construction expenditure is 611,998,960 NT$. NPV is 38,515,374, IRR is 10.27%, and PB is 29 years. Case 4 is dormitory with 2,840 beds. Construction expenditure is 815,153,627 NT$. NPV is 56,929,512, IRR is 10.42%, and PB is 29 years. The results are shown in the Table 1.

Table 3 List of parameters and financial indices of 4 cases

Parameters	Unit	Case 1	Case 2	Case 3	Case 4
Bed numbers	bed	710	1,420	2,130	2,840
Construction cost	NT$	205,689,625	408,844,293	611,998,960	815,153,627
Unit construction cost	NT$/Ping	86,816	86,281	86,103	86,014
NPV	NT$	1,687,097	20,101,236	38,515,374	56,929,512
IRR	%	9.16	9.99	10.27	10.42
PB	year	38	31	29	29

5.4 The calculation of the real options in every scenario

We have to determine the values of five parameters for the calculation of real option. The notation V_0 is the project NPV for every case. K=V_T is the project value at the end of project life. The term $\boldsymbol{\gamma_f}$ is 2%, which is the average rate of return of government bonds in 2006-2011. The σ is determined by Monte Carlo simulation. The time span, T, is 38 years. (see Table 2)

Table 4 List of parameters of real option model

Parameters	Values in this study	Definitions in the model
$\boldsymbol{V_0}$	NPVo	Current project value
$\boldsymbol{K \equiv V_T}$	(0, 0.5, 1, 1.5, 2) times of NPVo	Expected end project value
$\boldsymbol{\gamma_f}$	2%	Riskless rate of return %
$\boldsymbol{\sigma}$	$\log(\frac{NPV_0}{NPV_T})$	Standard error of the rate of return of NPV
T	38 years	Project life

To estimate the value of σ is very critical in calculating the real option. We need to carry out the sensitivity analysis in advance to find the critical factors of the project. Then, the range of change of every critical factors is determined by the market conditions, such as ratio of residence, inflation rate, maintenance fee, rate of rent, etc.. Finally, a Monte Carlo simulation is conducted by the assumed value of every critical factors. The results of the analysis is shown in Table 3.

Table 5 List of project values and $\boldsymbol{\sigma}$ of 4 cases

Case	NPVo (unit: NT$)	E(NPV) Expected NPV (unit: NT$)		Expected $\mathbf{log}(\frac{NPV_0}{NPV_T})$	
		E	Std	E	Std
Case1 710	1,687,097	4,214,161	17,966,990	2.49788	10.64965
Case2 1,420	20,101,236	25,157,400	35,513,040	1.25154	1.76671
Case3 2,130	38,515,374	46,297,750	53,080,020	1.20206	1.37815
Case4 2,840	56,929,512	66,480,110	71,045,390	1.16776	1.24795

With these 5 variables in every case, we can calculate the real option value of every case. The results are shown in Table 4.4.

Table 錯誤! 所指定的樣式的文字不存在文件中。.6 The real option value of 4 cases with various exercise value

	Vo	Ratio	K	C	C/Vo
Case 1 (710)	1,687,097	0.001	1,687	1,686,557	1.000
	1,687,097	0.5	843,549	1,417,316	0.840
	1,687,097	1	1,687,097	1,148,291	0.681
	1,687,097	1.5	2,530,646	888,859	0.527
	1,687,097	2	3,374,194	657,432	0.390
	Vo	Ratio	K	C	C/Vo
Case 2 (1,420)	20,101,236	0.001	20,101	20,101,236	1.000
	20,101,236	0.5	10,050,618	16,886,873	0.840
	20,101,236	1	20,101,236	13,680,989	0.681
	20,101,236	1.5	30,151,854	10,586,111	0.527
	20,101,236	2	40,202,472	7,833,099	0.390
	Vo	Ratio	K	C	C/Vo
Case 3 (2,130)	38,515,374	0.001	38,515	38,503,056	1.000
	38,515,374	0.5	19,257,687	32,356,430	0.840
	38,515,374	1.0	38,515,374	26,214,763	0.681
	38,515,374	1.5	57,773,061	20,292,094	0.527
	38,515,374	2.0	77,030,748	15,031,249	0.390
	Vo	Ratio	K	C	C/Vo
Case 4 (2,840)	56,929,512	0.001	56,930	56,911,305	1.000
	56,929,512	0.5	28,464,756	47,825,987	0.840
	56,929,512	1	56,929,512	38,746,473	0.681
	56,929,512	1.5	85,394,268	29,981,347	0.527
	56,929,512	2	113,859,024	22,184,432	0.390

Next, we carry out the sensitivity study of the real option. This study is to understand the parameters in real option model on how to influence the real option value. We have to set the $\boldsymbol{V_0}$. Figure 4.3 shows the change of the real option value with various V_T. Normalized real option value with various V_T is shown in Figure 4.2.

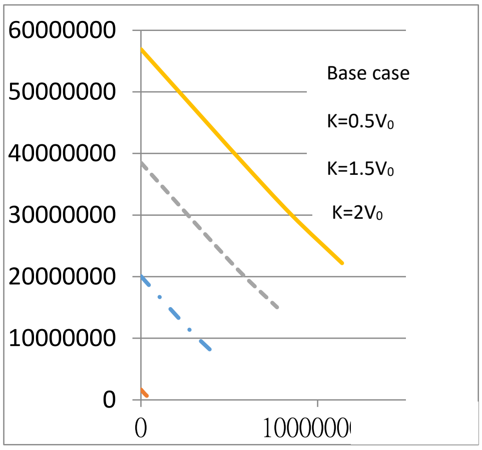


Figure 5.3 The real option value with various K

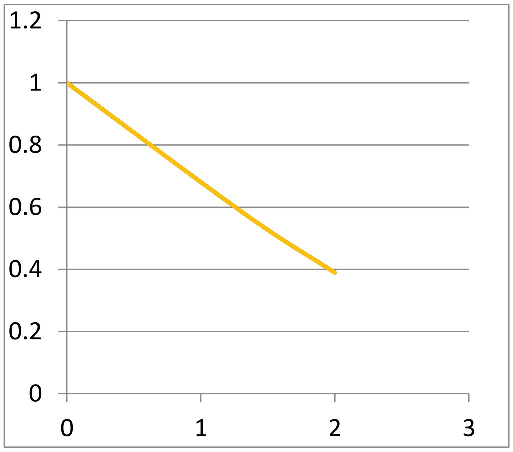


Figure 5.4 Normalized C with various K

Figure 4.5 shows the change of the real option value with various σ. Normalized real option value with various σ is shown in Figure 4.6.

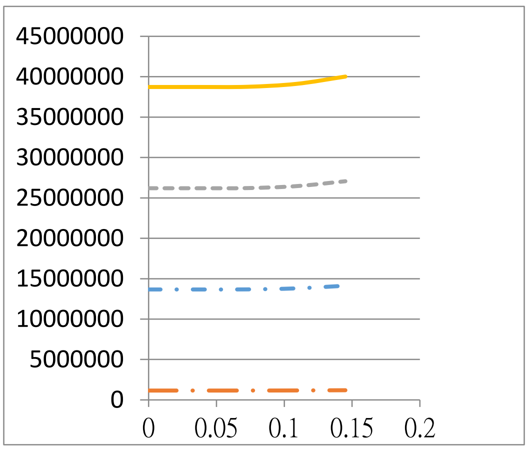


Figure 5.5 The real option with various σ

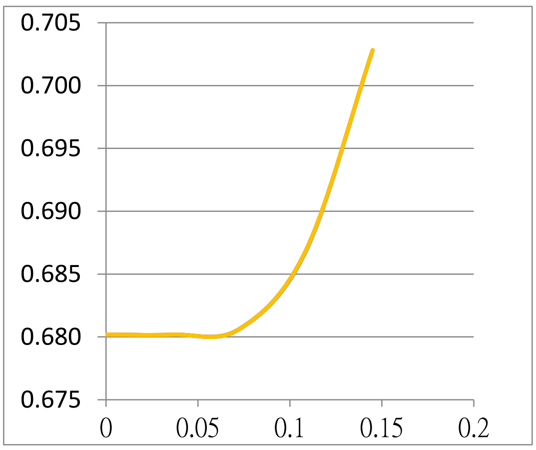


Figure 5.6 Normalized C with various σ

Figure 5.7 shows the change of the real option value with various γ_f. Normalized real option value with various γ_f is shown in Figure 4.8.

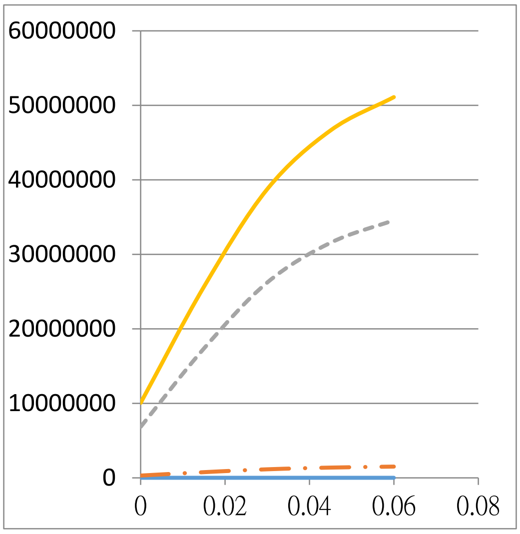


Figure 5.7 The real option with various γ_f

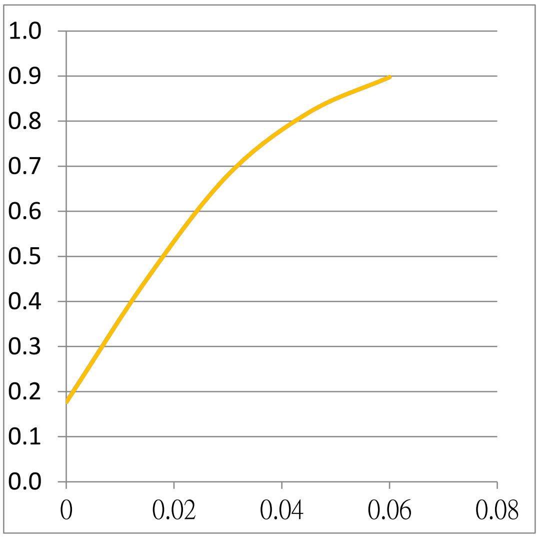


Figure 5.8 Normalized C with various γ_f

Figure 4.9 shows the change of the real option value with various T. Normalized real option value with various T is shown in Figure 4.10.

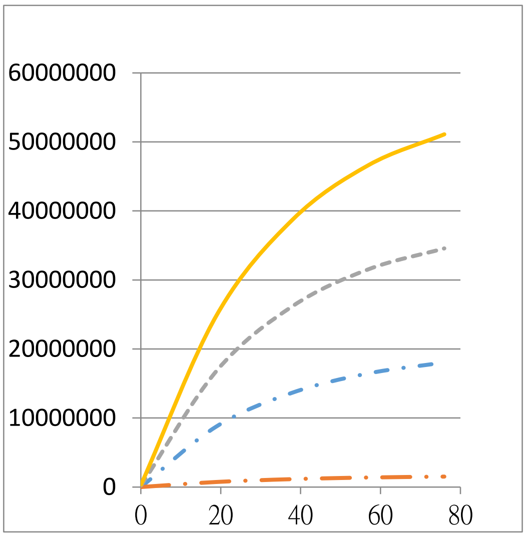


Figure 5.9 The real option with various T

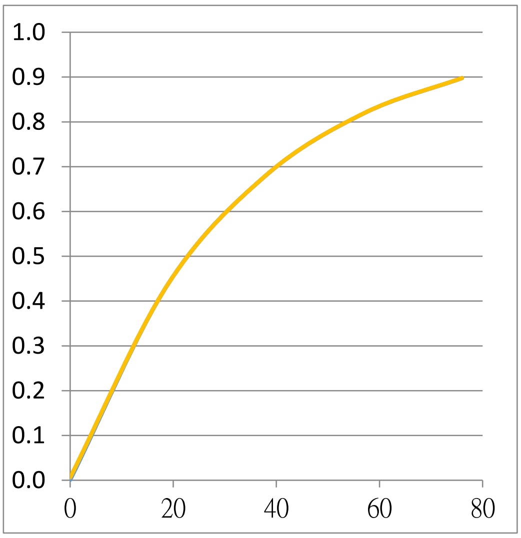


Figure 5.10 Normalized C with various T

5.5 Sensitivity analysis of real option and the calculations of Δ, θ, Γ, ν, ρ

Δ is to show the variation of project value on the value of real option. θ is to demonstrate the variation of project period on the value of real option. Γ is the second order differentiation of real option value on the project value. It is to reflect the rate of change of project value on the value of real option. ν is used to find the variation of σ on the value of real option. ρ is to find the variation of riskless interest rate on the value of real option. Table 5 shows the results of sensitivity analysis of the real option values.

Table 7 The sensitivity analysis of the real option

Case	Δ	θ	Γ	ν	**ρ**
1	0.9972	-391,981	0.0000013	10,371,193	20,503,497
2	0.9972	-4670,335	0.00000011	123,569,544	241,856,262
3	0.9972	-591,716	0.000000058	236,767,889	463,413,513
4	0.9972	-874,615	0.000000039	349,966,234	684,970,764

5.6 The sensitivity analysis with fixed end project value

In case of project value V0 is greater than K, it is called within value ($V_0/K>1$). This is the case for investors to exercise their right. In case of project value V_0 is equal to K, it is called even value ($V_0/K=1$). This is the case for investors to may exercise their right or may not. In case of project value V0 is smaller than K, it is called within value ($V_0/K<1$). This is the case for investors not to exercise their right. Figures 4.11-4.15 shows the results of the sensitivity analysis of real option with fixed end project value.

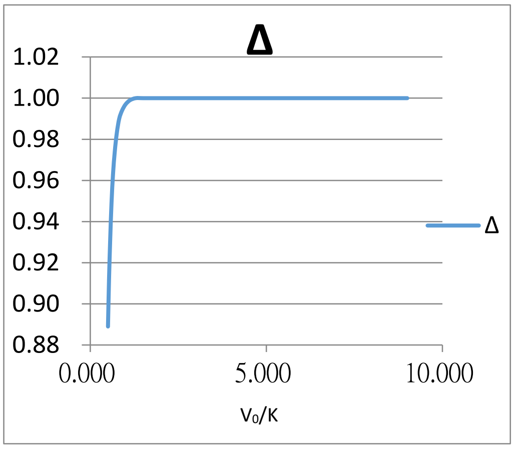


Figure 5.11Δ with various V_0/K

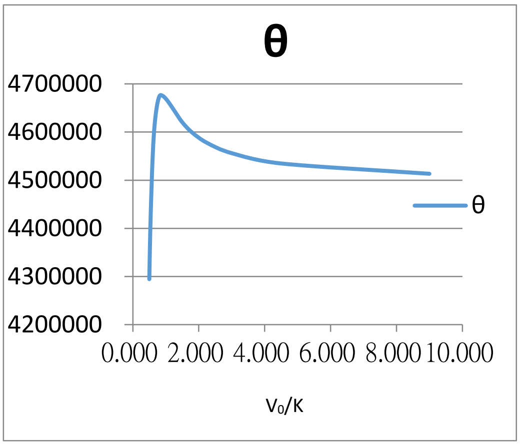
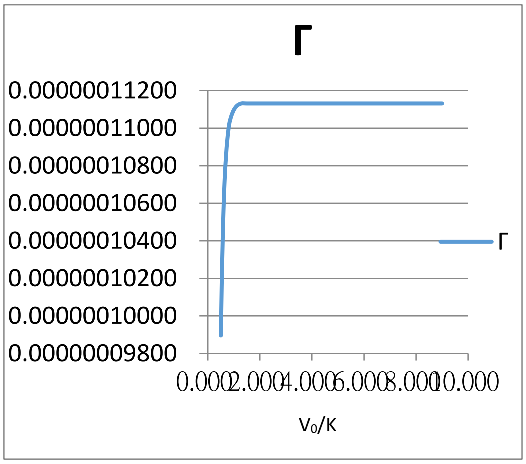


Figure 5.12 θ with various V_0/K



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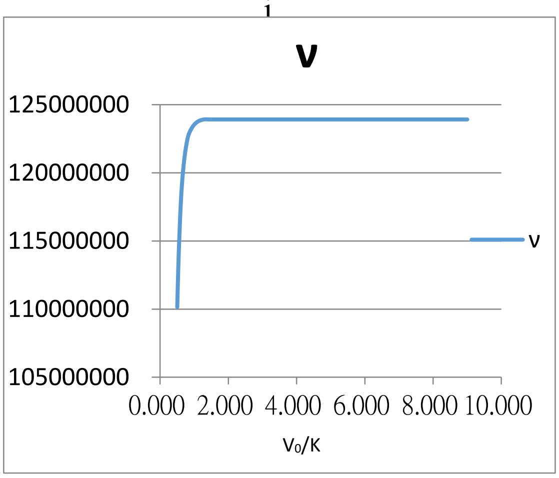

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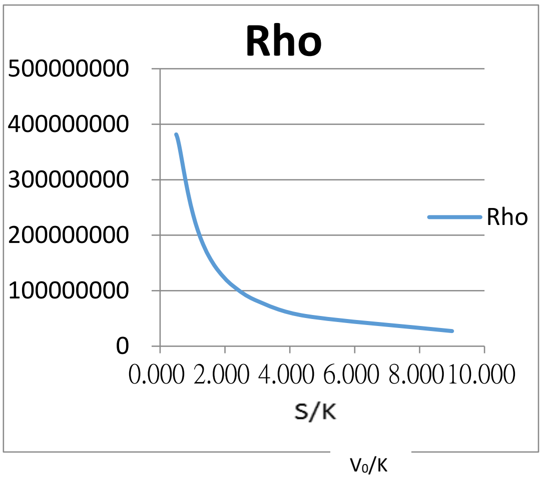

Figure SEQ Figure * ARABIC \

1. Δ (delta) is a hedge ratio in call option. Δ (delta) is a value between 0 to 1. It means that the real option value increases when the project value increases. The maximum value of the real option is equal to the current project value.
2. θ (Theta) increases when V_0/k increases. We find that θ (Theta) reaches maximum value when $V_0/K=1$.
3. Γ (Gamma) increases when 5V_0/K increases. The maximum value Γ (Gamma) occurs at $V_0/K=1$. .
4. ν (Vega) reflects the variation of project value on the value of real option. We find that ν (Vega) increases when V_0/K increases.
5. ρ (Rho) is to value the impact of riskless interest rate on the value of the real option. We find that ρ (Rho) decreases when V_0/K increases.

6. Conclusions

The analysis results reveal that the larger project the better financial results, higher NPV and IRR value. The sensitivity analysis in project financial analysis shows the interest rate of loans, inflation rate, and ratio of residence are the critical factors in the dormitory project. The results of Monte Carlo simulations shows that the variance of the growth rate of project value is quiet high, which means the project is with high risk in market perspective. It shows that this outcome may be the reason why not so many investors in favor of investing dormitory project in current market conditions.

Even this project bearing with high risk, results of the real option analysis suggest that the client may consider to provide the investors the managerial flexibility in the project. This managerial flexibility may contribute to enhance 68.1% of NPV values.

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